Cover page

Title: *Damage Identification in Beams by Piezodiagnostics for* Second European Workshop on Structural Health Monitoring

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ABSTRACT

This paper presents a numerical approach to damage identification based on the phenomenon of elastic waves propagation. The theoretical background is the dynamic Virtual Distortion Method (VDM). The signal perturbations due to damage in the structure are modelled by the so-called virtual distortions. The perturbed response is superposed over the intact response to elastic waves in a selected time period and the damaged response of the structure is modelled in this way. The presented numerical study puts forward a proposition how to identify damaged locations in beams in two stages. The first stage takes advantage of the VDM-based sensitivity analysis to point out the excitation frequency, best suited for further analysis. The second stage is actual dynamic inverse analysis using the Gradient Projection Method (GPM) as an optimisation tool. The problem of false damage locations, sometimes detected by the identification procedure, is discussed.

BACKGROUND - PIEZODIAGNOSTICS

The phenomenon of elastic wave propagation in structures has been utilized for parameter identification since the early 90's (cf. [1], [2]). The elastic waves are induced by piezoelectric transducers (PZTs), acting as actuators and generating the excitation, in some part of the structure. They are also captured by PZTs, acting as sensors able to detect the transient response, in some other part of the structure. The so-called structural signature, which is detected by sensors, is subsequently examined with respect to its variations due to structural damage. The intact structure response is compared with the damaged structure response. On that basis the identification procedure is able to determine the location (including extension) of damage in the structure and its intensity.

This paper is a continuation of research done within the PiezoDiagnostics (PD) project (cf. [3]), aiming at creating the integrated system for corrosion identification in engineering structures e.g. pipelines, cables, oil tanks. In the PD project, various

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laboratory demonstrators are used to prove the feasibility of the concept. The simplest of them is an aluminium beam, for which several experimental tests were carried out. The author will also use the beam model as a numerically simple, but sufficient to show the general idea of the method enabling damage identification.

FORMULATION OF THE DAMAGE IDENTIFICATION PROBLEM

Let us pose the optimisation problem of structural damage identification in the framework of the VDM (cf. [4]). We aim at minimising the following function:

$$\min f = \sum_A f_A = \sum_A \sum_t [d_A(t)]^2 \tag{1}$$

where:

$$d_A(t) = \varepsilon_A^M(t) - \varepsilon_A(t) = \varepsilon_A^M(t) - \left[\varepsilon_A^L(t) + \varepsilon_A^R(t)\right] = \varepsilon_A^M(t) - \sum_t \left[\Delta\varepsilon_A^L(t) + \Delta\varepsilon_A^R(t)\right] =$$

$$= \varepsilon_A^M(t) - \sum_{\tau \le t} \sum_{\tau' \le \tau} \left[\sum_\alpha D_{A\alpha}(\tau - \tau')\, \Delta\varepsilon_\alpha^0(\tau') + \sum_i D_{Ai}(\tau - \tau')\, \Delta\varepsilon_i^0(\tau') \right].$$

The function can be interpreted as an average departure of the total strain ε_A from the experimentally measured strain ε_A^M in locations *A*, capable of identifying the structural damage (i.e. *sensors*) in a selected period of time. Taking advantage of the VDM formulation (cf. [5], [6]), the strain ε_A is decomposed into two parts: ε_A^L, modelling the reference structural signature and ε_A^R, modelling perturbations to the reference signature due to damage. Both these parts are expressed as linear combinations of the influence matrix D components and virtual distortion ε^0 components (our design variable). The influence matrix D, storing the transient response in terms of strains, determines local-global inter-relations for the structure and is the basis for the VDM analysis. In the formula (1), the index *A* refers to the sensor, the index α to the actuator and the index *i* to the assumed damaged zone.

We shall measure the structural damage in each finite element with the help of the time-independent coefficient μ_i i.e. with the positive-valued ratio of cross-sectional area of a damaged element to the undamaged one. Having in mind the definition, we have to impose appropriate constraints on this coefficient. The initial value μ_i=1 refers to the intact structure. As we examine the physical process of deterioration of the element cross-section (e.g. due to corrosion), we are interested in such vector μ_i, which allows only for reduction of the initial cross-sectional area. On the other hand, only positive values of the vector μ_i may be considered in view of its definition (the value μ_i=0 refers to the completely damaged structure). Thus the constraints finally take the following form:

$$0 \le \mu_i \le 1 \quad i.e. \quad 0 \le \frac{\varepsilon_i(t) - \varepsilon_i^o(t)}{\varepsilon_i(t)} \le 1, \quad i = 1,\ldots,m. \tag{2}$$

The constraints (2), corresponding to the potentially damaged *m* finite elements, are non-linear with respect to the time-dependent design variable ε^0_i.

The advantage of the formulation is the possibility of calculating the gradients of the objective function, with respect to the coefficient μ_i, analytically. They are then used in the gradient-based optimisation performed in the identification process.

IDENTIFICATION PROCEDURE - DYNAMIC INVERSE ANALYSIS

In order to find the solution of the problem (1) subject to (2), it is necessary to perform the dynamic inverse analysis, employing one of the implicit integration algorithms e.g. the Newmark method.

It is also necessary to make use of an optimisation routine in order to find the minimum of the function (1). In this paper, the Gradient Projection Method (cf. [7], [8]) has been utilized as a constrained optimisation method. In the damage identification we look for a defect, whose extension is relatively small compared to the rest of the undamaged structure. Therefore in the optimal solution of this problem many constraints (2) turn out to be active. The GPM operates on active constraints and looks for the optimum in the subspace tangent to them. Therefore it is very well suited for the problem in question.

The cost of the inverse analysis grows rapidly with the number of the time steps and the number of finite elements suspected of damage. In order to reduce the cost, one should be precise with setting the compromise (accuracy vs. computational time) number of the time steps and also consider only the part of the structure most likely to be damaged. The latter information can be extracted by computing the initial gradients of the objective function with respect to the damage coefficient μ.

NUMERICAL EXAMPLES

Numerical model

The analysed numerical model is a 98 cm long cantilever beam, made of aluminium, discretised into 49 finite elements of 2 cm length. The Young's modulus is 65.8 GPa and the density - 2710 kg/m^3. The 10 first eigenfrequencies of the beam are listed in the TABLE I.

TABLE I. SELECTED EIGENFREQUENCIES OF THE ANALYSED BEAM

Eigenmode	Eigenfrequency [Hz]
1	4.1
2	26.0
3	72.7
4	142.5
5	235.5
6	351.8
7	491.2
8	653.9
9	839.8
10	1048.9

The beam is excited with a sine pulse of the frequency ω equal to the 7th eigenfrequency as the corresponding transient response (in 4/ω time) results in the biggest value of the objective function out of the 10 lowest eigenfrequencies. The eigenmode also provokes the most apparent variations in the initial gradient distribution, which highly influences the start of the identification procedure. In general, the higher the initial gradient, the more likely the element is to be damaged.

The damage is modelled numerically as the loss of stiffness executed by reducing the Young's modulus in a selected finite element.

Positioning of the actuator and sensor on the structure

SYMMETRICAL

First, the symmetrical mounting of the actuator and sensor on the beam was considered. The configuration with the actuator placed in the element No. 13 and the sensor in the element No. 37 was investigated. For this configuration the middle of the beam (element No. 25) coincides with the mid-distance between the actuator and sensor. The analysed transient response includes the waves reflected from boundaries (i.e. the clamped and free edge of the beam) and turns out to be identical for damage locations equally distant from the actuator or sensor if the distances of the actuator and sensor from the boundaries are the same (as in this configuration).

The effect of the specific mounting of the actuator and sensor on the beam is demonstrated in FIGURE 1. As the first case, the damage was assumed in the element No. 19 and the corresponding response is the curve marked by circles. As the second case, the damage was assumed in the element No. 31 and the response curve, marked by triangles, is identical with the previously generated one. Therefore for the symmetrical configuration, a false damage location (except for the true one) will be observed as a result of the identification process i.e. if damage is assumed only in the element No. 19, the identification algorithm will also detect it in the element No. 31 and vice versa. This is straightforward because the optimisation algorithm operates here on identical transient responses. In other words, the problem lacks uniqueness for this configuration of the piezodevices. FIGURE 2 depicts the result of the identification of damage, located in the element No. 19, of the intensity μ=0.5 (the position of the vertical axis corresponds to the middle of the beam). The algorithm indicates two damage locations of less severe intensity.

ASYMMETRICAL

Subsequently, the asymmetrical mounting of the actuator and sensor on the beam was considered. The configuration with the actuator placed in the element No. 19 and the sensor in the element No. 43 was investigated.

The effect of this mounting of the actuator and sensor on the beam is demonstrated in FIGURE 3. As the first case, the damage was assumed in the element No. 25 and the corresponding response is the curve marked by circles. As the second case, the damage was assumed in the element No. 37 and the response curve, marked by triangles, differs significantly from the previously generated one. On the contrary to the symmetrical configuration, the identification process detects the damage location precisely for each case (see FIGURES 4 and 5).

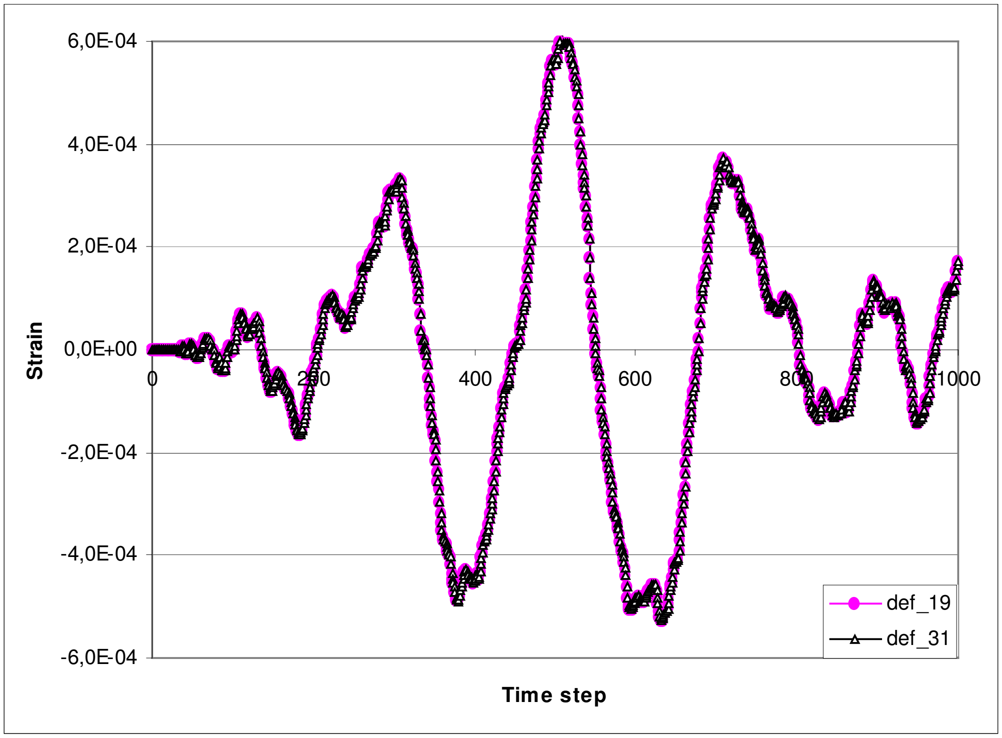

Figure 1. Transient responses for damage located in the elements Nos. 19 or 31.

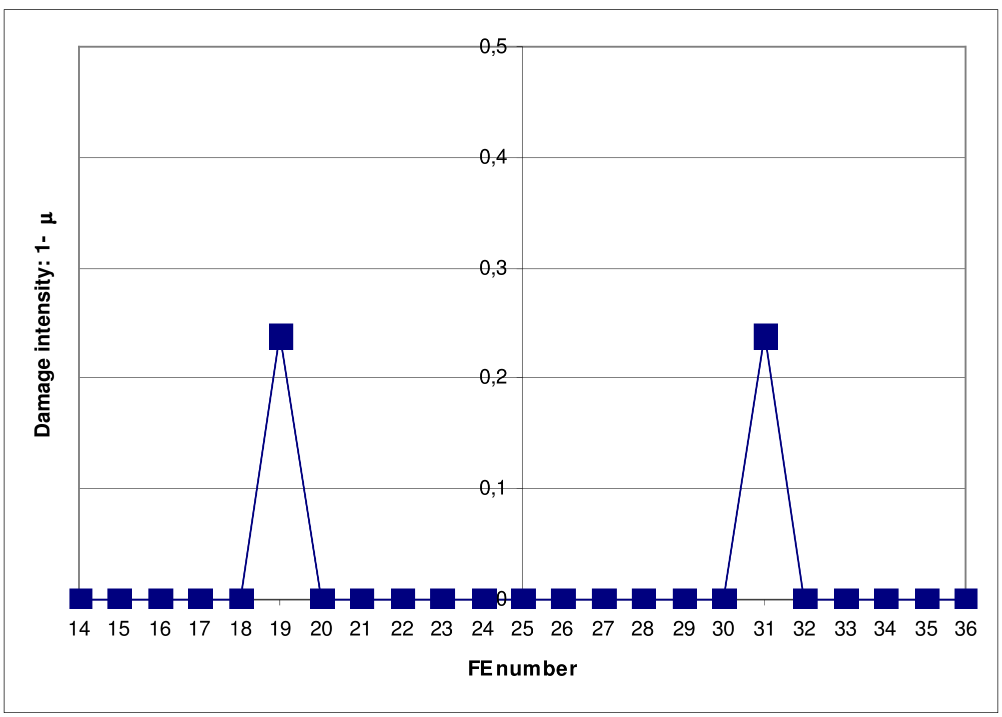

Figure 2. Damage identification for the symmetrical mounting (true damage in the el. No. 19).

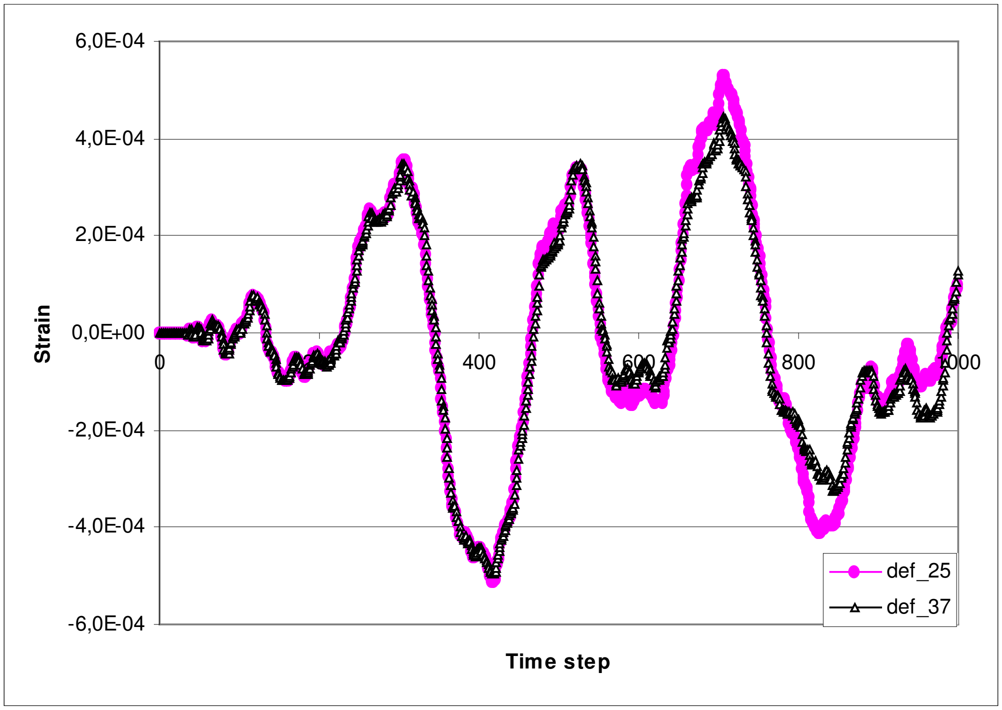

Figure 3. Transient responses for damage located in the elements Nos. 25 or 37.

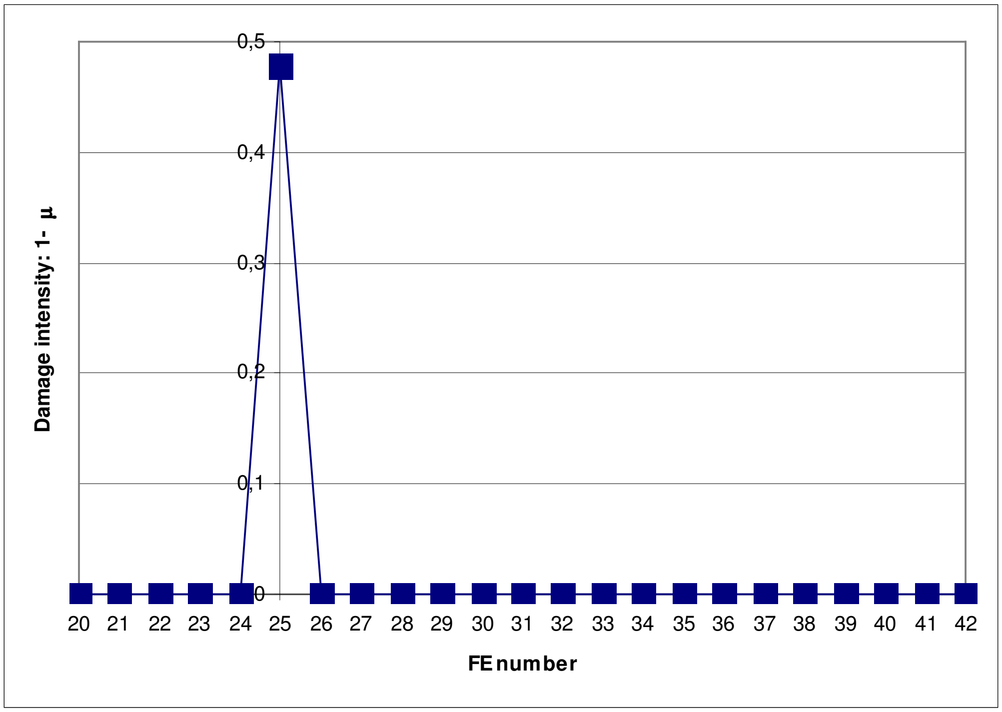

Figure 4. Damage identification for the asymmetrical mounting (true damage in the el. No. 25).

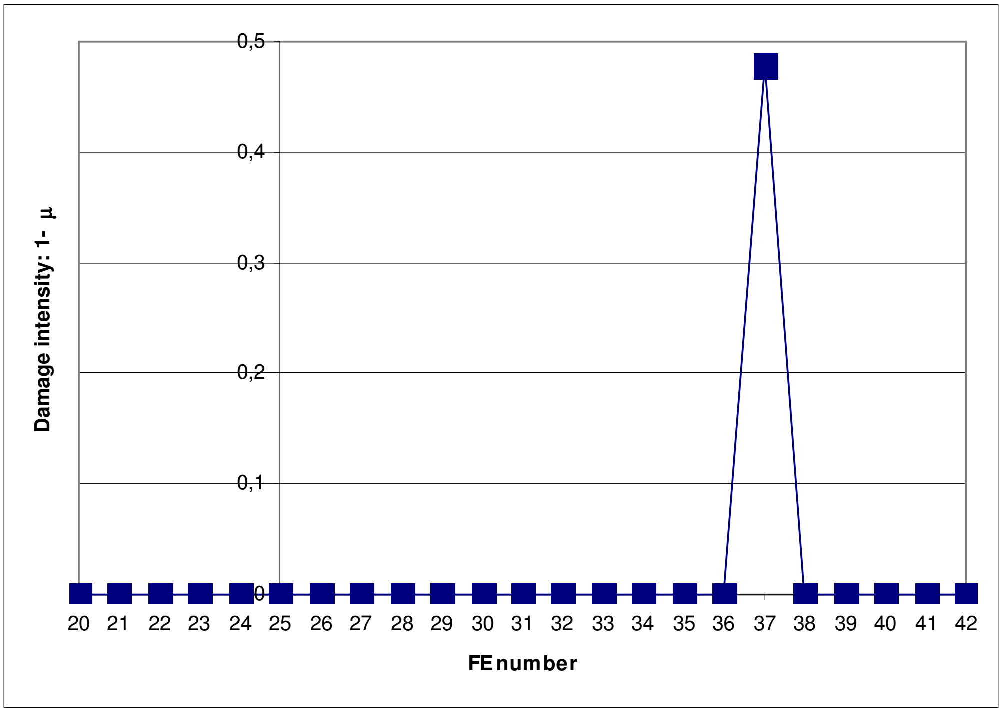

Figure 5. Damage identification for the asymmetrical mounting (true damage in the el. No. 37).

CONCLUSIONS

The paper presents a numerical study of the problem of damage identification in beams. No experimental data was included in the analysis thus the measured response ε_A^M appearing in the formula (1) should be regarded as noise-free.

The beam structure is excited with sine pulse of a frequency chosen on the basis of primary sensitivity analysis. The analysis is performed for a set of low frequencies (e.g. 10 lowest eigenfrequencies in order to ascertain the long-distance propagation of elastic wave) and determines one frequency, which results in the biggest objective function value and/or high variations of initial gradients. The sensitivity analysis is numerically inexpensive, but informative as it roughly indicates the potential locations of damage. It may also be utilised to limit the considered damaged zone to the finite elements of high gradients (not all elements between the actuator and sensor) and therefore to reduce the computational cost of the actual inverse analysis.

The identification procedure consists in performing the dynamic inverse analysis based on the Newmark method in the time domain. In order to find the minimum of the objective function (1), the Gradient Projection Method has been employed as an advanced optimisation method, taking care about the constraints (2). The GPM has proved to be effective in finding the optimal solution precisely (indicating the element in which the damage was primarily assumed), which was possible thanks to the numerical generation of the response of the damaged structure (noise-free ε_A^M).

The problem of false damage locations has been investigated. It has turned out that it is due to the specific mounting of the actuator and sensor on the structure. The analysed transient response includes elastic waves reflected from the boundaries. Therefore, if the distances of the actuator and sensor from the beam edges are the same, the reflected waves interfere so that two different damage locations (symmetrical with respect to the middle of the beam) provoke identical transient responses. This cannot be obviously overcome by the optimisation algorithm, which ends up with both true and false damage locations. The remedy for the problem is proper positioning of the actuator and sensor i.e. their distances from the boundaries of the structure should be apparently different.

Future research will focus on verifying the proposed method against experimental data (noisy ε_A^M). The reduction of numerical cost, which may be considerable for real engineering structures, will be investigated. The problem of false damage locations still remains a challenge for "infinitely" long structures, e.g. pipelines, where each relative position of the actuator and sensor is symmetrical with respect to the boundaries of the structure. It seems that two sensors, mounted on such a structure relatively close to each other, should distinguish between true and false damage locations. This will also be the subject of future work.

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